HYPERSONIC FLOW-PAST OF PLANE BLUNT BODIES BY A NONVISCOUS RADIATING GAS

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An analytic solution is obtained in the work in a Newtonian approximation [1] for the flow-past problem for a plane blunt body by a steady-state uniform hypersonic inviscous space-radiating gas flow. The hypersonic flow-past problem for axisymmetrical blunt bodies by a nonviscous space-radiating gas has been previously considered [2-4]. In this case a satisfactory solution of the problem was obtained even in a zero-th approximation by decomposing the unknown values in terms of a parameter ε equal to the ratio of gas densities before and after passage of the shock wave. The solution of the problem in a zero-th approximation with respect to ε in the case of flow-past of plane blunt bodies does not turn out to be satisfactory, since the departure of the shock and the radiant flux to the body as gas flows into the shock layer turns out to be strongly overstated under nearly adiabatic conditions. Freeman [5] demonstrated that results may be significantly improved for flow-past of a plane blunt body by a nonradiating gas if a more precise expression is used for the tangential velocity component expressed in a new approximation with respect to the parameter ε. This refinement is applied in this work for solving the flow-past problem for a plane blunt body by a space-radiating gas. The distribution of the gas-dynamic parameters in the shock layer, the departure of the shock wave, and the radiant heat flux to the surface of the body are found. The solution obtained is analyzed in detail for the example of flow-past regarding a circular cylinder.

1. The system of dimensionless equations describing the flow of a nonviscous nonheat-conducting, chemically balanced space-radiating gas in a shock layer about a plane blunt body has the form [1]

$$\rho\left(u\frac{\partial u}{\partial x}+\varepsilon^2 v\frac{\partial v}{\partial x}\right)=-\varepsilon\frac{\partial p}{\partial x}; \tag{1.1}$$

$$\frac{\varepsilon}{H}\frac{\partial v}{\partial x}-\frac{u}{RH}=-\frac{\partial p}{\partial\psi};$$

$$\frac{\rho u}{H}\frac{\partial}{\partial x}(h+u^2+\varepsilon^2 v^2)=-\Gamma K_P T^4;$$

$$\frac{\partial y}{\partial x}=H\frac{v}{u};\quad \frac{\partial y}{\partial\psi}=\frac{1}{\rho u};\quad H=1+\frac{\varepsilon y}{R(x)};$$

$$r(x,y)=r_w(x)+\varepsilon y\sin\alpha;$$

$$\Gamma=\frac{8K_{Ps*}\sigma T_{s*}^4\varepsilon l}{\rho_\infty V_\infty^3};$$

$$\varepsilon=\rho_\infty/\rho_{s*};\quad h=h(p,T);\quad \rho=\rho(p,T).$$

Here lx and $\varepsilon l y$ are coordinates directed along the surface of the body and along the normal to it, uV_∞ and $\varepsilon V_\infty v$ are the velocity components of the gas in the direction of these coordinates, $\varepsilon^{-}\rho_\infty\rho$ is the density, $\rho_\infty V_\infty^2 p$ is the pressure, $0.5V_\infty^2 h$ is the enthalpy, $T_{s*}T$ is the gas temperature, $K_{Ps*}K_P$ is the Planck absorption coefficient, $lR(x)$ is the radius of curvature of the surface of the body, lr is the distance from the axis of symmetry to a given point, l is the characteristic linear scale, Γ is the radiation parameter, α is the Stefan–Boltzmann constant, α is the angle between the tangent to the body and the direction of the veloc-

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ity of undisturbed flow, and $\rho_\infty V_\infty/\psi$ is a stream function determined by the equation $d\psi = \rho\, u dy - \rho v H dx$. The indices ∞, s, *, and w denote the incident flow parameters, parameters directly behind the shock wave, the characteristic values of the parameters, and their values on the body surface, respectively.

In deriving the system (1.1) it was assumed that self-radiation of the comparatively cold surface of the body can be ignored.

The boundary conditions on the shock wave have the form

$$\begin{gathered}\psi = \psi_s(x) = r_s(x) = r_w(x) + \varepsilon y_s \sin\alpha;\\ u_s(x, \psi_s) = \cos\beta\cos(\beta-\alpha) + \varepsilon\frac{\rho_{s*}}{\rho_s}\sin\beta\sin(\beta-\alpha);\\ v_s(x,\psi_s) = \cos\beta\sin(\beta-\alpha) - \varepsilon\frac{\rho_{s*}}{\rho_s}\sin\beta\cos(\beta-\alpha);\\ p_s(x,\psi_s) = (\gamma_\infty M_\infty^2)^{-1} + \left(1-\varepsilon\frac{\rho_{s*}}{\rho_s}\right)\sin^2\beta;\\ h_s(x,\psi_s) = h_\infty + \left(1-\varepsilon^2\frac{\rho_{s*}^2}{\rho_s^2}\right)\sin^2\beta;\\ \mathrm{tg}(\beta-\alpha) = \varepsilon y'_{sx}/(1+\varepsilon y_s/R),\end{gathered} \tag{1.2}$$

where β is the angle between the tangent to the shock wave and the direction of undisturbed flow, M_∞ is the Mach number of the incident flow, and $y_S = y_S(x)$ is the shock-wave equation. We assume that

$$\psi=0 \tag{1.3}$$

on the surface of the body.

2. The solution of the system of equations (1.1) with boundary conditions (1.2), (1.3) is found in the form of decompositions [1, 5] with respect to a small parameter ε,

$$f(x,\psi,\ \varepsilon)=f_0(x,\ \psi) + \varepsilon f_1(x,\ \psi)+\ldots, \tag{2.1}$$

where f is any of the functions u^2, v^2, t, ρ, p, h, or T. It is further assumed that all these functions and their first derivatives are magnitudes on the order of one in the shock layer. Substituting the decomposition of Eq. (2.1) in the system of equations (1.1), we obtain to a zero-th approximation,

$$\begin{gathered}\frac{\partial u_0}{\partial x} = 0;\ \frac{\partial p_0}{\partial\psi} = \frac{u_0}{R};\ \rho_0 u_0\frac{\partial h_0}{\partial x} = -\Gamma K_P T_0^4;\\ \rho_0=\rho_0(p_0,\ T_0);\ h_0=h_0(p_0,\ T_0);\ r=r_w(x).\end{gathered} \tag{2.2}$$

We will further limit ourselves to the zero-th approximation (2.2) for h_0, p_0, and T_0 following previous works [5, 6], and we will use for the tangential and normal velocity components and the geometric coordinates, the first-order approximation

$$\begin{gathered}\frac{\partial u_1^2}{\partial x} = -\frac{2}{\rho_0}\frac{\partial p_0}{\partial x};\quad u^2 = u_0^2 + \varepsilon u_1^2;\\ \frac{\partial y}{\partial\psi} = \frac{1}{\rho_0 u},\quad v = u\frac{\partial y}{\partial x}.\end{gathered} \tag{2.3}$$

The boundary conditions in the shock take the form

$$\begin{gathered}\psi=\psi_s(x)=r_w(x);\ u_{s0}(x,\ \psi_s)=\cos\ \alpha(x);\ p_{s0}(x,\psi_s)=\sin^2\alpha(x);\\ h_{s0}(x,\psi_s)=\sin^2\alpha(x);\ u_{s_1}(x,\psi_s)=0.\end{gathered} \tag{2.4}$$

Integrating the first equations of (2.2) and taking into account Eqs. (2.4), we obtain

$$u_0(x,\ t)=\cos\alpha(t); \tag{2.5}$$

$$p_0(x,t) = \sin^2\alpha(x) - \frac{1}{R(x)}\int_t^x \cos\alpha(t)\sin\alpha(t)\,dt, \tag{2.6}$$

where t is a coordinate fixing the point at which a given streamline enters the shock layer.

An analysis of tables [7, 8] demonstrates that in definite temperature and pressure intervals we may use the equation of state (in dimensional form)

$$h=\gamma/(\gamma-1)p/\rho \tag{2.7}$$

(γ is the effective ratio of the heat capacities of the gas in the shock layer) and also approximates the Planck absorption coefficient in the form

$$K_P=ApT^n, \tag{2.8}$$

where A and n are the constants of approximation. Then the energy equation, taking into account Eqs. (2.4)-(2.8), has the solution

$$\begin{gathered} h_0(x, t)=\left\{\sin^{-2(n+4)}\alpha(t)+\frac{b(x-t)}{\cos\alpha(t)}\right\}^{-\frac{1}{n+4}}; \\ b=8A\rho_\infty V_\infty^2\left(\frac{V_\infty^2}{2C_p}\right)^{n+4}\varepsilon l\left(\frac{\gamma+1}{2\gamma}\right)\frac{\sigma(n+4)}{\rho_\infty V_\infty^3}. \end{gathered} \tag{2.9}$$

We find the solution of the system of equations (2.3), taking into account Eqs. (2.5)-(2.9),

$$\begin{gathered} u^2(x, t)=\cos^2\alpha(t)-\varepsilon\left(\frac{\gamma+1}{\gamma}\right)\times\int_t^x\frac{dx' h_0(x', t)}{p_0(x', t)}\frac{\partial p_0}{\partial x'}(x', t); \\ y(x, t)=\int_0^t\frac{\sin\alpha(t)\,dt}{\rho_0(x, t)\,u(x, t)}. \end{gathered} \tag{2.10}$$

Departure of the shock wave is determined from Eqs. (2.10) at t = x. The normal velocity component is found by differentiating Eq. (2.10): $v = u\partial y/\partial x$.

We determine the distribution of the radiant flux along the surface of the body on the basis of the parameters obtained for the gas flow in the shock layer,

$$q_w(x)=\frac{2q_R(x)}{\rho_\infty V_\infty^3}=\frac{b}{2(n+4)}\int_0^x\frac{h^{n+5}(x,t)\sin\alpha(t)\,dt}{u(x,t)}.$$

3. We will consider in more detail, as an example, flow-past of a circular cylinder of radius R by a hypersonic air flow. In this case we obtain, given the geometric relations $l = R$, $\alpha(x) = \pi/2 - \theta$; and $\Phi = \pi/2 - \alpha(t)$,

$$\begin{gathered} u_0(\Phi)=\sin\Phi; \\ p_0(\theta, \Phi)=\cos^2\Phi-\frac{\sin^2\theta}{2}+\frac{\sin^2\Phi}{2}; \\ h_0(\theta, \Phi)=\left\{\cos^{-2(n+4)}\Phi+\frac{b(\theta-\Phi)}{\sin\Phi}\right\}^{-\frac{1}{n+4}}. \end{gathered} \tag{3.1}$$

We note that the method of solution is not suitable in a neighborhood of a Newtonian point of discontinuity, at which pressure at the surface of the body falls to zero [5]. For a cylinder this corresponds to a value of the angle θ_* calculated at the critical point,

$$\theta_*=\arcsin\sqrt{2/3}=54°44'.$$

The tangential velocity component for the first-order approximation has the form

$$u^2(\theta, \Phi)=\sin^2\Phi-\varepsilon\left(\frac{\gamma+1}{\gamma}\right)\int_\Phi^\theta d\theta' h_0(\theta', \Phi)\frac{\partial\ln p_0(\theta', \Phi)}{\partial\theta'}. \tag{3.2}$$

We represent departure of the shock wave by

$$y_s(\theta) = \frac{\gamma+1}{2\gamma} \int_0^1 \frac{dt h_0(\theta, \Phi(t)) \sin\theta}{\left[\cos^2\theta - \frac{\sin^2\theta}{2}(1-t^2)\right] u(\theta, \Phi(t))}, \tag{3.3}$$

where a substitution of variables in the form $\sin\Phi = t \sin\theta$, has been carried out. Then the radiant flux to the surface of the body is determined by the equation

$$q_w(\theta) = \frac{b \sin\theta}{2(n+4)} \int_0^1 \frac{dt h_0(\theta, \Phi(t))^{n+5}}{u(\theta, \Phi(t))}. \tag{3.4}$$

4. Let us consider the limiting cases of the equations we have obtained. We approximately set in the integrand of Eq. (3.2)

$$h_0(\theta', \Phi) \simeq h_0(\theta, \Phi). \tag{4.1}$$

This substitution introduces an error only in the terms $0(\varepsilon)$ and allows us to obtain a correct result in the case of the flow of a nonradiating gas. Equation (3.2) is integrated, taking into account Eq. (4.1),

$$u^2(\theta, \Phi) = \sin^2\Phi - \varepsilon\left(\frac{\gamma+1}{\gamma}\right) h_0(\theta, \Phi) \ln\left[\frac{p_0(\theta, \Phi)}{\cos^2\Phi}\right].$$

At $b \ll 1$, when the gas radiates weakly, Eq. (3.1) implies

$$h_0(\theta, \Phi) = \cos^2\Phi. \tag{4.2}$$

Taking into account Eqs. (3.3) and (4.2), we obtain the equation for the departure of the shock wave,

$$y_s(\theta) = \frac{\gamma+1}{2\gamma} \sin\theta \int_0^1 \frac{dt\,(1 - t^2\sin^2\theta)}{\left[\cos^2\theta - \frac{\sin^2\theta\,(1-t^2)}{2}\right]\sqrt{F(\theta, t)}};$$

$$F(\theta, t) = t^2\sin^2\theta - 2\varepsilon(1 - t^2\sin^2\theta) \ln\left(\frac{\cos^2\theta - \frac{\sin^2\theta}{2}(1-t^2)}{1 - t^2\sin^2\theta}\right). \tag{4.3}$$

We find the value for the angles $\theta \leq \pi/6$ for which we decompose in Eqs. (4.3) the logarithmic function in a series in powers of $\sin^2\theta$ (cancelling by $\sin\theta$) and omit in the integrand remaining terms on the order of $\sin^2\theta$ and above,

$$y_s(\theta) = \frac{\gamma+1}{2\gamma} \frac{1}{\cos^2\theta} \frac{1}{\sqrt{1-3\varepsilon}} \ln\left(\frac{1+\sqrt{1-3\varepsilon}}{\sqrt{3\varepsilon}}\right). \tag{4.4}$$

We obtain from Eq. (4.4) on the critical line ($\theta = 0$) at $3\varepsilon \ll 1$ an equation for departure of the shock wave (in dimensional form)

$$\Delta_s(0) = \frac{\varepsilon R}{2} \ln\frac{4}{3\varepsilon}.$$

This equation agrees with previous results [5, 6] obtained for the flow of a nonradiating gas.

The radiant flux to the surface of the cylinder at $b \ll 1$ has the form

$$q_w(\theta) = \frac{b \sin\theta}{2(n+4)} \int_0^1 \frac{dt\,(1 - t^2 \sin^2\theta)^{n+5}}{\sqrt{F(\theta, t)}}. \quad (4.5)$$

The same flux is calculated at the critical point $\theta = 0$ exactly by

$$q_w(0) = \frac{b}{2(n+4)} \frac{1}{\sqrt{1-3\varepsilon}} \ln\left(\frac{1+\sqrt{1-3\varepsilon}}{\sqrt{3\varepsilon}}\right).$$

The physical meaning of this equation is that it gives a radiant flux from a homogeneous plane, space-radiating gas layer with thickness equal to the magnitude of the shock-wave departure (4.4). Carrying out the decompositions similar to those used in the derivation of Eq. (4.4), from Eq. (4.5) we obtain

$$q_w(\theta) \cong q_w(0) \cos^m\theta; \quad (4.6)$$

$$m = (n+5)\left[\frac{\sqrt{1+a}}{\ln\left(\frac{1+\sqrt{1-3\varepsilon}}{\sqrt{3\varepsilon}}\right)} - a\right];$$

$$a = 3\varepsilon/(1-3\varepsilon).$$

We may note that, unlike the analogous expression obtained for sphere flow-past [3], the exponent m in Eq. (4.6) turns out to be less and depends not only on n, but also on ε. Moreover, the magnitude of the radiant flux at the critical point $q_w(0)$ is greater for a cylinder than for a sphere due to the greater magnitude of shock-wave departure. We will take $\varepsilon = 0.05$ and $n = 8$ for a numerical estimate on the basis of the tables [7, 8]. In this case the ratio of the magnitudes for the cylinder (index 1) to the magnitude for a sphere (index 2) is written in the form

$$q_{w1}(0)/q_{w2}(0) = 1.74; \; m_1/m_2 = 0.748.$$

In a second limiting case, when the gas is strongly radiating ($b \gg 1$), the asymptotic calculation of the integral in Eq. (3.4) for angles $\theta \leq \pi/6$ leads to the equation for shock-wave departure,

$$y_s(\theta) = \frac{\gamma+1}{2\gamma} \frac{n+4}{n+3} \frac{b^{-\frac{1}{n+4}}}{\cos^{2-\frac{1}{n+4}}\theta}. \quad (4.7)$$

The distribution of the radiant flux at $b \gg 1$ under these assumptions has the form

$$q_w(\theta) = \frac{\cos^3\theta}{2}. \quad (4.8)$$

A comparison of the equations for a cylinder (4.4) and (4.6)-(4.8) with the corresponding equations previously obtained [3] for sphere flow-past demonstrate that the strongest difference in the radiant flux $q_w(\theta)$ and the shock-wave departure $y_s(\theta)$ is observed for flow-past of these bodies by a weakly radiating gas ($b \ll 1$).

The magnitudes $q_w(\theta)$ and $y_s(\theta)$ for flow-past of a plane and axisymmetrical body are similar in the case of a strongly radiating gas ($b \gg 1$). This is explained by the fact that the distribution of the flow parameters near the shock wave, and not in a sublayer near the body, where the behavior of the parameters is less important for determining $q_w(\theta)$ and $y_s(\theta)$, plays a determining role at $b \gg 1$.

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